Introduction:

In this investigation I have chosen to look at how the temperature of water affects the buoyant force exerted on objects submerged in water. To do this, I have created a simple model using a piece of cork attached to a force meter and held under water in a water-boiler while it heats up. I devised a formula for this relationship by combining Archimedes' principle and the equation of volumetric thermal expansion of liquids.

Research question:

What is the relationship between the temperature of water and the buoyant force exerted on an object submerged in the water?

Independent variable: Temperature of the water (T)

Dependent variable: Buoyant force (F)

Controlled variables:

- Volume of water displaced (V)
- Initial temperature of water (T_0)
- Initial pressure of water (p_0)
- Weight of the cork
- Gravitational acceleration (g)

Theory:

Archimedes' principle states that the buoyant force of an object submerged in water equals the weight of water displaced by the object[1].

Because of Newton's second law of motion, we know that

$F = m * a$

Density equals mass divided by volume.

$p = \frac{m}{V} \Rightarrow m = p * V$

$F = p * V * g$

This means that the buoyant force is proportional to the density of the water, assuming the volume displaced (V) and acceleration (g) are constants.

$F \propto p$

[1] Encyclopedia Britannica. (2019). *Archimedes' principle* | *Description & Facts*. [online] Available at: https://www.britannica.com/science/Archimedes-principle [Accessed 3 Apr. 2019].

Furthermore, we can use the equation for volumetric expansion of liquids[2]

$$\Delta V = V_0 * \beta * \Delta T$$

where ΔV is the volume change of the total amount of liquid in the container, V_0 is the initial volume of liquid, β is the coefficient of volumetric thermal expansion, and ΔT is the temperature change.

Density equals mass divided by volume.

$$p = \frac{m}{V} \Rightarrow V = \frac{m}{p}$$

The volume change equals current volume minus initial volume.

$$\Delta V = V_1 - V_0$$

Combined, this means that

$$\Delta V = \frac{m}{p} - \frac{m}{p_0}$$

$$\frac{m}{p} - \frac{m}{p_0} = \frac{m}{p_0} * \beta * \Delta T \quad \rightarrow \quad \frac{m*p_0}{p*m} - \frac{m*p_0}{p_0*m} = \beta * \Delta T$$

$$\frac{p_0}{p} - 1 = \beta * \Delta T \quad \rightarrow \quad \frac{p_0}{p} = \beta * \Delta T + 1$$

$$\frac{1}{p} = \frac{\beta*\Delta T+1}{p_0} \quad \rightarrow \quad p = \frac{p_0}{\beta*\Delta T+1}$$

By substituting for p, we get this equation

$$F = \frac{p_0}{\beta*\Delta T+1} * V * g$$

Assuming beta (β) and initial density (p_0) are constant, the equation can be written

$$F = k * \frac{1}{\Delta T+k}$$

Which means the buoyant force is inversely proportional to the the temperature change plus a constant k.

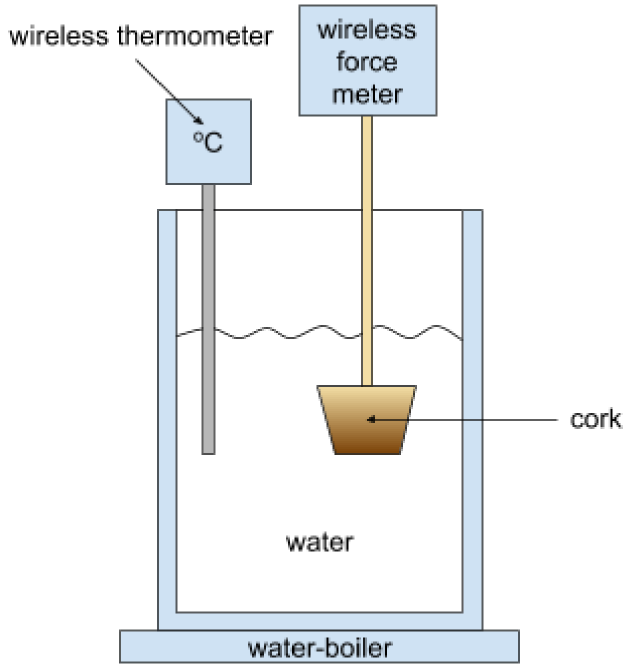
$$F \propto \frac{1}{\Delta T+k}$$

Hypothesis:

From the theory, I can deduct that the buoyant force is inversely proportional to the temperature change plus a constant. This means that when the temperature increases, the buoyant force will decrease.

[2] Elert, G. (2019). *Thermal Expansion – The Physics Hypertextbook*. [online] The Physics Hypertextbook. Available at: https://physics.info/expansion/ [Accessed 30 Mar. 2019].

Method:



The experiment was set up as shown in the figure above.

Measurement of the temperature of water:

After the water-boiler was filled with 1.5 liters of water of a temperature around 9ºC, it was turned on, allowing the temperature to gradually rise as the heater did its job. This increase in temperature was measured using a PASCO wireless temperature meter connected to PASCO Capstone where the data was collected.

Measurement of the buoyant force:

A piece of cork was held underwater, exerting an upwards force on the stick holding it down. This force was measured using a PASCO wireless force acceleration meter, also connected to PASCO Capstone for data collection. Choosing the same rate of measurement (1 Hz) allowed for temperature and force to be measured simultaneously and logged in Capstone. To find the buoyancy, the weight of the cork (0.30 N) was subtracted from the measurements of force.

Controlling the controlled variables:

One issue with using the water-boiler was that bubbles formed. In the initial experiments, this affected the force, especially when the water was nearing 100ºC and starting to boil. To avoid this, I opted for turning the boiler off at regular intervals while it was heating, letting the bubbles disappear, and then taking the measurements.

Another issue was controlling the weight of the piece of cork submerged. If placed in the water while dry, the small air-pockets in the cork would gradually fill with water and change the weight throughout the experiment. To solve this, I began soaking the cork in water before starting my experiment.

Range of independent variable:
Doing the measurements in intervals meant I had to create a new data run in Capstone for each interval, and altogether this made for 20 different runs. To narrow down my data collection, I then picked out the three first measurements for each run, totaling in 60 measurements of both the independent and dependent variable.
The minimum temperature of the water was as cold as I could get it from the sink, which was 9.5^{o}C in my final runthrough of the experiment. The maximum temperature was as close to boiling I could get without burning my hand, which was holding the force meter, and was 99.1^{o}C. The other 58 measurements are all in between 9.5^{o}C and 99.1^{o}C.
As the independent variable in my theory is not temperature, but change of temperature, I chose 9.5^{o}C as my starting temperature, and subtracted this value from all measurements of temperature. The range of my independent variable is therefore temperature change between 0^{o}C and 89.6^{o}C.

Results:

Force/N ±0.01	Temp./C ±0.1
0.51	9.5
0.51	14.5
0.52	19.0
0.52	21.4
0.52	25.9
0.52	32.3
0.52	36.8
0.54	44.4
0.53	50.7
0.54	50.8
0.54	54.8
0.55	60.0
0.56	66.8
0.57	72.3
0.60	79.0
0.63	83.9
0.67	90.7
0.70	93.8
0.69	96.0
0.73	99.1

This table does not show all 60 measurements, but a selection of one measurement from each of the 20 intervals.

The uncertainty of the upwards force measured is ±0.01 N since the smallest digit on the force meter used was 0.01 N. The uncertainty of the temperature measured is ±0.1°C since the smallest digit on the thermometer used was 0.1°C.

Analysis:

According to my theory, the buoyant force is inversely proportional to the change of temperature plus one over beta (ΔT+1/β).

$$F = \frac{p_0 * V * g}{\beta * \Delta T + 1}$$

This can be graphed as a rational function f(x)=A/(Bx+1).
A would equal $p_0 * V * g$ and B would equal β.

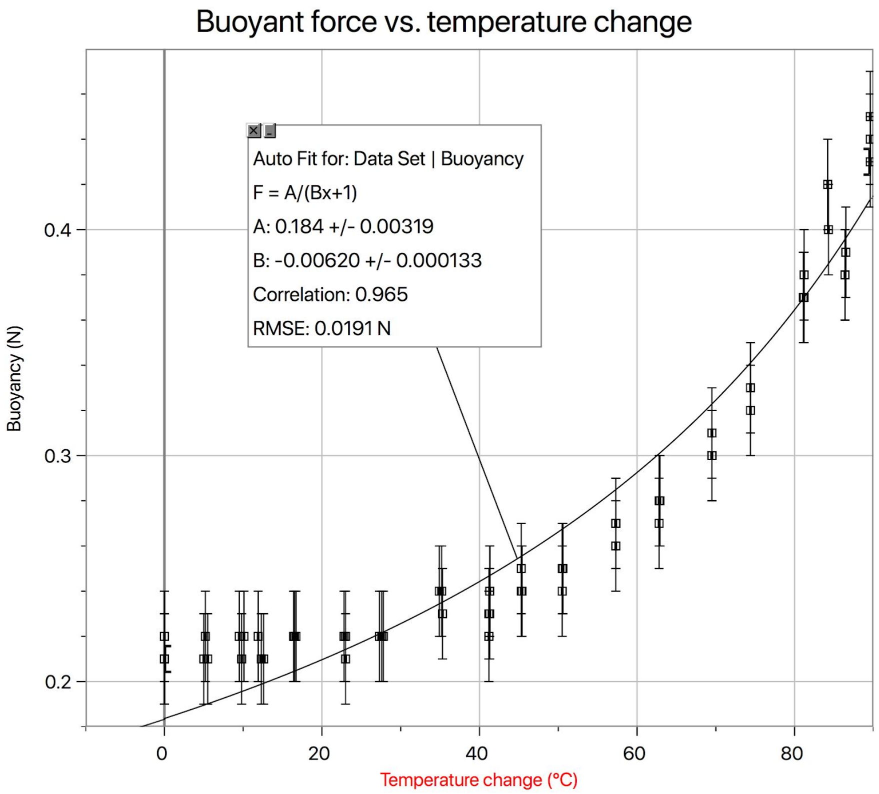
My current data is showing the measurements of upwards force and the temperature. In my theory I am using buoyant force and temperature change, so I have to process my data to get the right values.

To find buoyant force, I subtract the weight of the cork (0.30 N) from each value of force measured. As both the upwards force and the weight have an uncertainty of ±0.01 N, the buoyant force will then have an uncertainty of the sum of the two uncertainties; ±0.02 N.

To express the temperature as temperature change instead, I subtract my starting temperature (9.5°C) from each measurement of temperature. The uncertainty for temperature change will then be ±0.1°C.

Force/N ±0.01	Temp./C ±0.1	Weight/N ±0.01	Buoyancy/N ±0.02	Temp. change/C ±0.1
0.51	9.5	0.30	0.21	0.0
0.51	14.5		0.21	5.0
0.52	19.0		0.22	9.5
0.52	21.4		0.22	11.9
0.52	25.9		0.22	16.4
0.52	32.3		0.22	22.8
0.52	36.8		0.22	27.3
0.54	44.4		0.24	34.9
0.53	50.7		0.23	41.2
0.54	50.8		0.24	41.3
0.54	54.8		0.24	45.3
0.55	60.0		0.25	50.5
0.56	66.8		0.26	57.3
0.57	72.3		0.27	62.8
0.60	79.0		0.30	69.5
0.63	83.9		0.33	74.4
0.67	90.7		0.37	81.2
0.70	93.8		0.40	84.3
0.69	96.0		0.39	86.5
0.73	99.1		0.43	89.6

Graph of F vs ΔT:



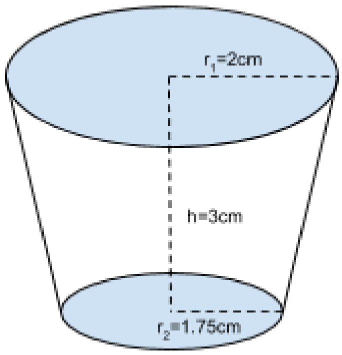
The value of A is 0.184 N ±0.00319 and the value of B is -0.00620°C^{-1} ±0.000133.

Conclusion:

Per my calculations, both A and B have 1/β as a factor.

$A = p_0 * V * g$ $\qquad\qquad B = \beta$

Using the values of A and B from the graph, I can work out the values of β and p_0, but first the values of V and g must be defined.



The piece of cork used for my experiment had the shape of a truncated cone with the measurements as shown on this figure. The volume of this shape is

$$\frac{\pi*r_1^2+\pi*r_2^2}{2} * h \quad \rightarrow \quad \frac{\pi*(0.02m)^2+\pi*(0.018m)^2}{2} * 0.03m$$

which is $3.41*10^{-5}$ m^3.

The value of g is estimated to 9.81 ms^{-2}.

$p_0 = \frac{A}{V*g}$

p_0 is then equal to 549.75 kgm^{-3} ±30.92.

β is -0.00620.

The problem here is that the value of β is negative, which would mean the volume of water is decreasing as the temperature rises, and the buoyant force is then becoming greater at higher temperatures.

My graph shows a clear trend of the buoyant force increasing as the temperature change is increasing. This contradicts what was stated in my theory, which said that buoyancy should decrease as temperature change increased.

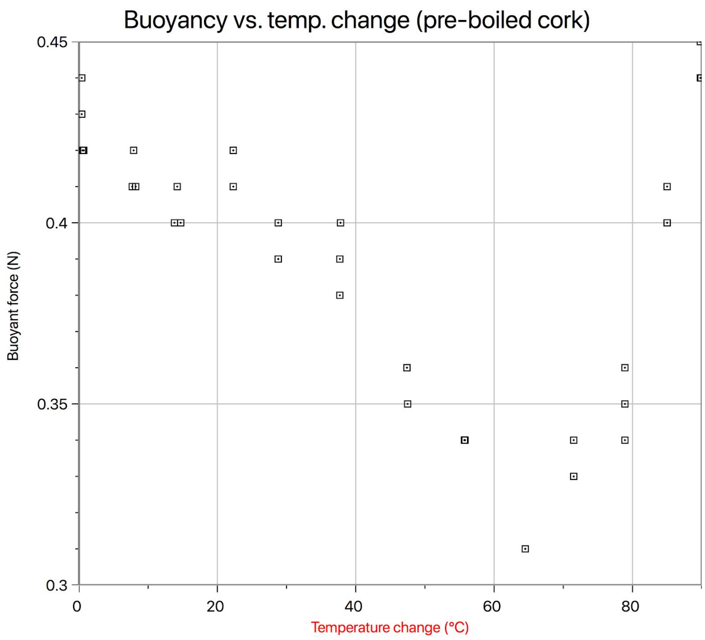
I can therefore not conclude that my hypothesis is correct, as my data does not support it.

Discussion and evaluation:

In my theory, I made the assumption that the volume displaced by the piece of cork was constant. This is a potential source of inaccuracy, as most materials experience some form of volume change due to temperature. If the volume of the cork expands more than water does, this could potentially offset my results.

Another source of possible errors is that the force meter holding the piece of cork was handheld, and movement caused by this can have been cause of random errors or variations in measurements. This could be improved by elevating the force meter over the water-boiler by either attaching popsicle sticks across the top of the water-boiler that the force meter could rest on or by elevating it using a clamp stand. Not having the force meter handheld would also have increased the safety of my experiment, as I was in risk of burning my hand on the steam rising from the water-boiler as it heated up and started boiling.

To try and avoid the inaccuracy caused by the cork expanding, I repeated my experiment the same way as before, but I pre-boiled the cork to have the starting volume be the same as the volume when most expanded.



The graph seemed to go the right way until the temperature change became greater than 65°C and the buoyant force started to rise again. The reason for this is probably because the cork was shrinking as it gradually reached the same temperature as the water, and then started to expand again once the temperatures were leveled. Another issue with cork is that some water is absorbed, and this can contribute further to the expansion of the material as the temperature rises. The popsicle stick connecting the cork to the force meter had some buoyancy of its own, which could also have offset my results.

To avoid both of these problems, I repeated the experiment using a material that would not absorb any water. Instead of cork, I used a piece of brass. As the weight of the brass was larger than its buoyancy it didn't float, so there was no need to hold it down with the popsicle stick. I attached it with a thin metal string instead, and the buoyant force experienced on this string is so small as to be negligible.

Assuming the volume displaced by the piece of brass will expand as temperature increases, I can add on to my formula for buoyant force.

$F = \frac{p_0}{\beta * \Delta T + 1} * V * g$

Volumetric thermal expansion can be expressed using the linear expansion coefficient ɑ

$\Delta V = 3\alpha * V_0 * \Delta T$ [3]

Substituting with V - V_0 for ΔV gives $V = V_0 * (3\alpha * \Delta T + 1)$

Combining these formulas, the buoyant force can be expressed as

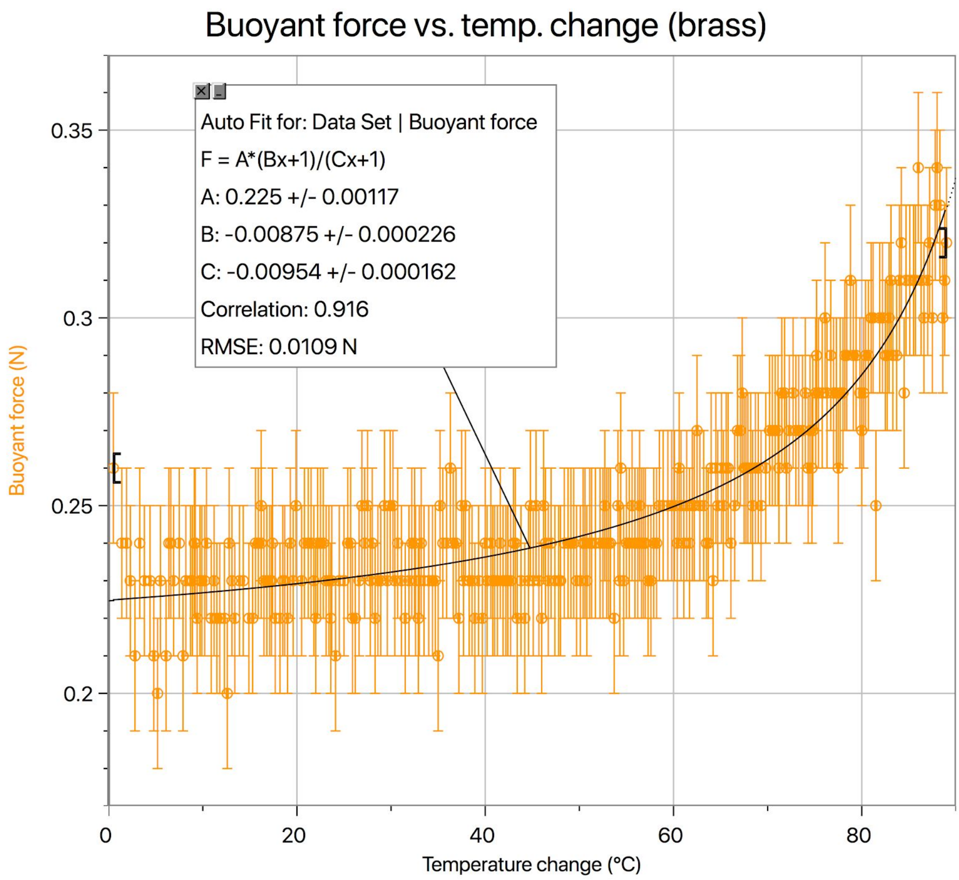
$F = \frac{p_0}{\beta * \Delta T + 1} * V_0 * (3\alpha * \Delta T + 1) * g$

$F = \frac{k*(k\Delta T + 1)}{k*\Delta T + 1}$ can be expressed using the rational function f(x)=A(Bx+1)/(Cx+1).

A would then be $p_0 * V_0 * g$, B would be 3α, and C would be β.

ɑ and β can be expressed using B and C

$\alpha = B/3$ $\beta = C$



By substituting the values of B and C, this gives ɑ = -0.00292 and β = -0.00954.

Again this makes for the same problem; if the coefficients of thermal expansion are negative, that means the volume should be decreasing as temperature increases, meaning the

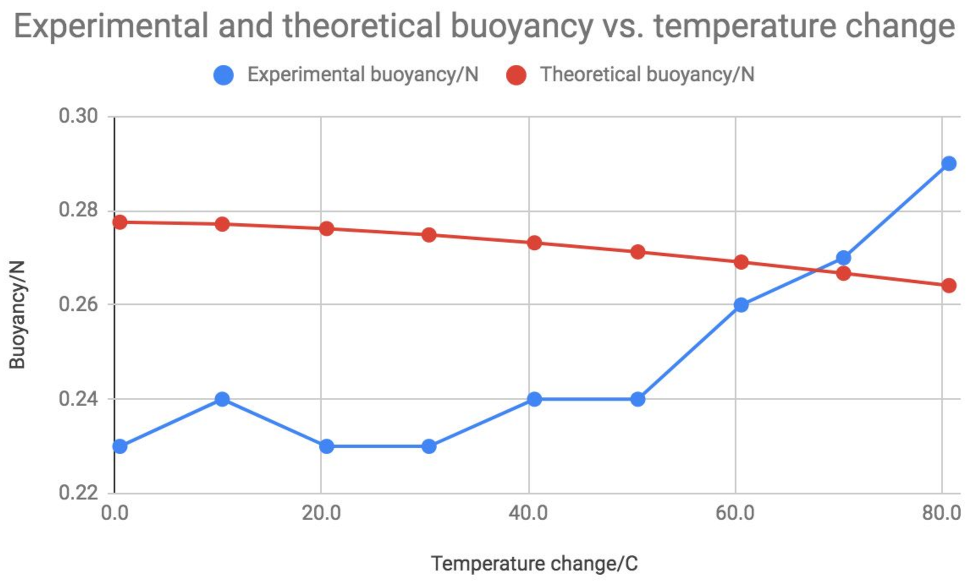
[3] Elert, G. (2019). *Thermal Expansion – The Physics Hypertextbook*. [online] The Physics Hypertextbook. Available at: https://physics.info/expansion/ [Accessed 30 Mar. 2019].

buoyant force increases, which again goes against my hypothesis. The graph shows a clear trend of the buoyant force increasing as temperature increases.

Another assumption made in the theory is that β is a constant. According to published values [4], β goes from $88*10^{-6}$ $^{o}C^{-1}$ at 10°C to $695*10^{-6}$ ^{o}C at 90°C, which is an almost 700% increase and far from being a constant. Knowing these values, I can make a graph showing both my experimental values and the theoretical ones I can now calculate using the varying values of β.

Theoretical temp./C	Beta/C^-1	Actual temp./C	Experimental buoyancy/N	Temp. change/C	Theoretical buoyancy/N
10	0.000088	10.0	0.23	0.5	0.278
20	0.000207	19.9	0.24	10.4	0.277
30	0.000303	30.0	0.23	20.5	0.276
40	0.000385	39.9	0.23	30.4	0.275
50	0.000457	50.1	0.24	40.6	0.273
60	0.000522	60.1	0.24	50.6	0.271
70	0.000582	70.1	0.26	60.6	0.269
80	0.000640	80.0	0.27	70.5	0.267
90	0.000695	90.2	0.29	80.7	0.264
Initial density/kgm^-3	Initial volume/m^3	Gravity/ms^-2	Alpha/C^-1		
999.77	0.0000283	9.81	0.0000203		

α is the linear coefficient of thermal expansion for brass ($20.3*10^{-6}$ $^{o}C^{-1}$)[5]
The initial density is 999.77 kgm^{-3} at 10 ^{o}C[6].

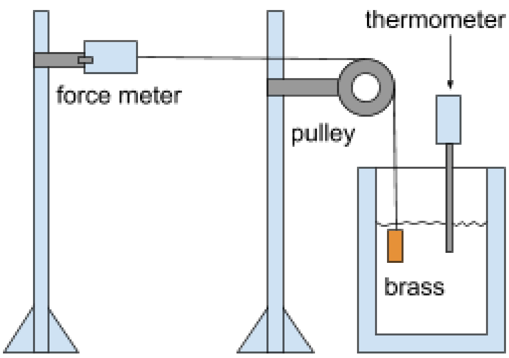


[4] Elert, G. (2019). *Thermal Expansion – The Physics Hypertextbook*. [online] The Physics Hypertextbook. Available at: https://physics.info/expansion/ [Accessed 30 Mar. 2019].
[5] Elert, G. (2019). *Thermal Expansion – The Physics Hypertextbook*. [online] The Physics Hypertextbook. Available at: https://physics.info/expansion/ [Accessed 30 Mar. 2019].
[6] *Density of liquid water from 0°C to 100°C.* [online] VaxaSoftware. Available at: http://www.vaxasoftware.com/doc_eduen/qui/denh20.pdf [Accessed 29 Mar. 2019].

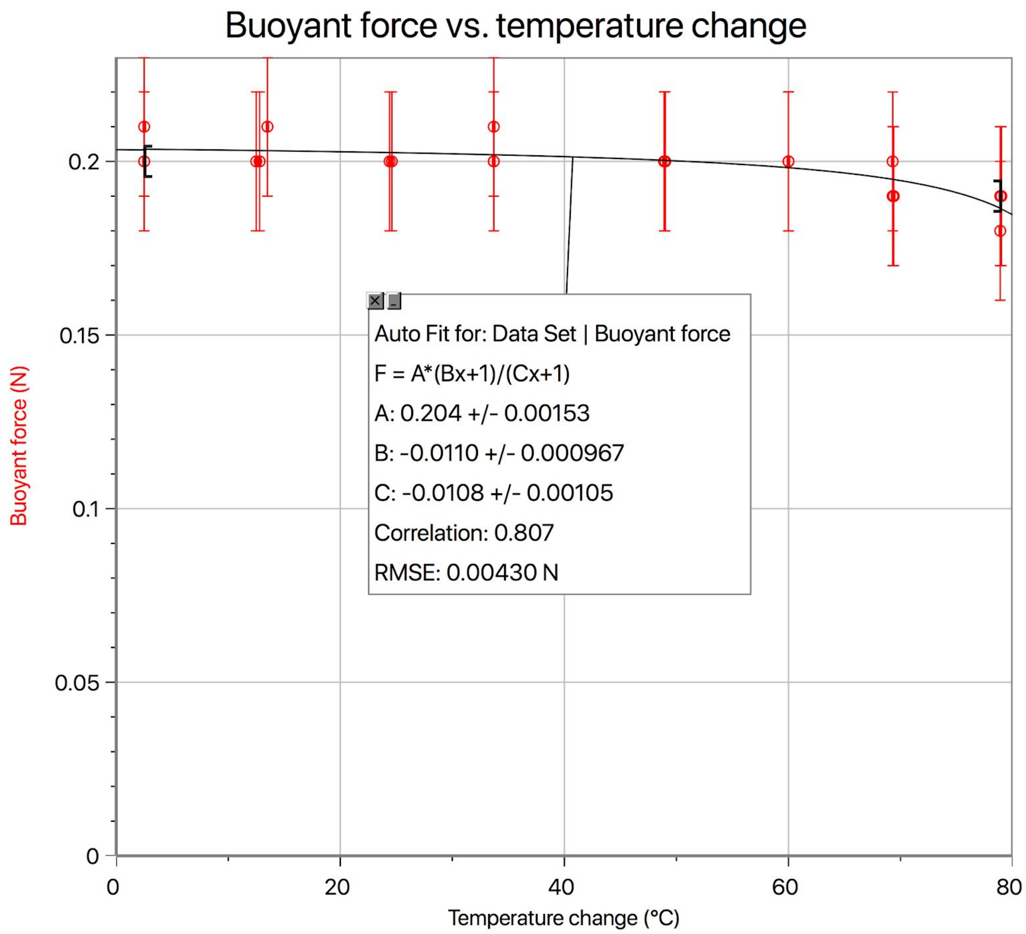
As my data still does not fit my theory, even with the theory adjusted to make up for changing values of β, there has to be some source of error. A possibility is that my measuring equipment is not working correctly. The thermometer should be fine, as it is meant to endure high temperatures. The force meter, however, is not intended to be held right over boiling water. If the device was affected by being heated up too much, this could be a possible explanation as to why my data did not fit. Doing a simple experiment to test this out, I zeroed the measurements on the force meter and held it above the water boiler without anything attached. It started as zero, but when the temperature rose to around 70°C the measurements started increasing. When the water boiled, the force measured had increased to 0.15 N (starting from 0.00 N).

After contacting PASCO Technical Support, I was informed that the intended functional temperature range of the force meter was from 0°C to 40°C and that at higher temperatures the resistance of the strain gauge inside the force meter was affected, which caused the errors of measurement.

As my data had rather small margins and increases, this inaccuracy of measurement makes a really big difference and is likely the cause of why my data does not fit. What could be done to improve this is to either hold the force meter further up and away from the water, or create a pulley system so the force meter can be held to the side of the water-boiler and still measure the force accurately enough.



I repeated my experiment with the addition of a pulley, set up like the figure above. Attaching the force meter to a clamp stand also removed errors caused by my hand moving while holding it.



This alteration seems to have made a big difference, as the graph is now headed in the right direction; showing that the buoyancy decreases with a temperature increase. However, the uncertainties for buoyancy are very large compared to the spread of data, likely because the piece of brass used had a small volume so the change in buoyancy was fairly small and could not be measured accurately enough with the equipment at hand. Repeating the experiment on a larger scale using a piece of brass or other material with a larger volume, and hence a larger change in buoyancy, could show the relationship between temperature change and buoyant force more accurately.

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Elert, Glenn. "Thermal Expansion – The Physics Hypertextbook". *The Physics Hypertextbook*, 2019. https://physics.info/expansion/.