# Physics IA report

## Introduction
In this investigation, I will analyze the relationship between the chamber volume of a plastic bottle and the frequency of the sound produced when air is blown into it at an angle.

The plastic bottle used for the experiment is a 0.65L Imsdal bottle (Norwegian brand). It has the shape of a cylinder and has a short bottle neck and small opening:

## Research question
When blowing air into a bottle at an angle, what is the relationship between the volume of the chamber and the frequency of the sound produced?

Dependent variable
- Frequency

Independent variable
- Volume of air (changed by adding water increments)

Controlled variables
- Measuring equipment
- Plastic water bottle (cylinder shape)
- Environment (temperature, altitude, sunlight etc)
- Water temperature

## Theory

Definition of Sound waves: “a wave of compression and rarefaction, by which sound is propagated in an elastic medium such as air.” This means that sound waves are the propagation of changing pressures (Physics HL - Chris Hamper - Second Edition - Pearson 2014). When air is blown into a bottle the air pressure inside is increased. The air particles blown in cause a disturbance of pressure so that the air in the bottle compresses. This can be compared to a oscillating spring. The compression of a spring is similar to the compression of the air in the bottle. The tension of the spring/air will be small and the resultant force will be upwards out of the bottle. All the energy has one into compressing the air will be released, thus the air decompresses and escapes the bottle. This oscillating process will repeat itself as long as air is blown into the bottle at an angle.The sound wave travelling into the bottle will be reflected in the bottom. The reflected wave and the original wave superpose to give a standing wave.

The produced sound is due to the Helmholtz resonance. The frequency produced can be described by it:

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{Vl}}$$

$$A = area\ of\ port\ opening$$

$$v = speed\ of\ the\ sound = c = 343ms^{-1}$$

$$V = volume\ of\ air\ in\ the\ chamber$$

$$l = length\ of\ the\ opening\ port$$

**Deducting Helmholtz resonance theory**

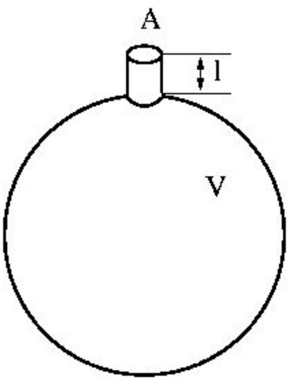
Helmholtz resonance is based on Helmholtz equation. Helmholtz observed that the volume of a resonator is inversely proportional to the square frequency of the sound:

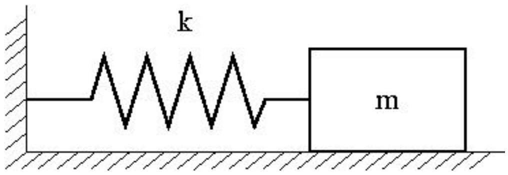
$$V\,\alpha f^2$$

or:

$$f^2 V = constant$$

As mentioned before, the comparison to a oscillating spring proves helpful for the rest of the deduction:




The mass (m) puts pressure on the spring - likewise the air in the neck puts pressure on the air in the resonant chamber. The mass of the neck can be described as:

$$\rho A l = m$$

$$\rho = density\ of\ air$$
$$l\ = neck\ length$$
$$A = area\ of\ the\ opening$$

The air pressure can be compared with the spring constant *k*. The air pressure constant can be found as (equation borrowed from helmholtz's volume resonator - Kamaljeeth Instrument):

$$k = \rho c^2 \frac{A^2}{V}$$

$$c = velocity\ of\ the\ speed\ of\ sound$$

Now assuming the principle the mass on a spring is similar to air pressure in a bottle, it is possible to use the spring-mass system equation to get Helmholtz resonance:

Spring-mass equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

T can also be written as 1/f, giving:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If we then insert the air pressure constant and the air mass:

$$f = \frac{1}{2\pi}\sqrt{\frac{\rho c^2 \frac{A^2}{V}}{\rho A l}}$$

Rearranging:

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{V\,l}}$$

However there might be some implications with the formula.

- The equation does not take into account the geometric figure of the bottle. (will be discussed later on).
- The neck only takes into account the physical length, while the effective length might be different. This is due to the boundary conditions - air can escape efficiently from the opening with only a tiny change in pressure. The frequency then becomes damped making it hard to define the frequency and/or volume.

There is an correction formula that gives a more accurate theoretical value, thus closing the gap between the theoretical and actual values:

$$l' = l + 1,5r$$

Were:

$$r = radius\ of\ port\ opening$$

Thus:

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{V\,l'}}$$

Correction found at: http://www.vibrationdata.com/Newsletters/January2004_NL.pdf

## Hypothesis

My hypothesis is that the frequency produced will increase for each addition of water, as the volume of air then decreases. This should be the case according to Helmholtz equation where:

$$f \propto \sqrt{\frac{1}{V}}$$

Therefore I expect the results to form a linear graph that goes through origo.

## Method

The experiment will require the use of an audio recording program (Google science journal), a plastic bottle, a graduated measuring cylinder for the water amounts and a microphone.

Order of procedure:

1. For this experiment, we can assume that the room temperature and speed of sound are standard.
2. The microphone should be placed by the opening port to best record the sound.
3. Starting with a empty bottle, I will blow air into the bottle at an angle so a clear sound is produced. This sound is recorded and noted down with Google Science Journal.
4. The bottle can then be filled with an increment of water, thereby decreasing the volume of air in the chamber. A new recording can take place with the same method as before.
5. This process will be repeated with successive additions of water until a satisfactory amount of data has been recorded.

How the variables will be controlled:
- Measuring equipment - the same measuring equipment will be used during the whole experiment.
- Plastic water bottle (cylinder shape) - the same plastic water bottle will be used for all measurements.
- Environment (temperature, altitude, sunlight etc) - the experiment will be performed in the same environment for the whole duration.
- Water temperature - the water temperature will be room temperature by using water from the same source system (like a jug of water)

**Results**

| Volume water/ml ±0.5ml | Volume air/ml ± 0.5ml | Mean frequency/Hz |
|---|---|---|
| 0 | 650 | 181,4 |
| 50 | 600 | 186,3 |
| 100 | 550 | 200,7 |
| 150 | 500 | 212,6 |
| 200 | 450 | 227,7 |
| 250 | 400 | 237,5 |
| 300 | 350 | 249,3 |
| 350 | 300 | 269,9 |
| 400 | 250 | 285,6 |
| 450 | 200 | 327,3 |
| 500 | 150 | 370,6 |

For measuring the water, I used a 100ml graduated cylinder. The smallest unit of measurement on the cylinder was 1ml, therefore the uncertainty is:

$$\frac{1ml}{2} = 0.5ml$$

To figure out the uncertainty of the frequency, the data has to be manipulated, this is explained in the analysis part.

**Analysis**

Google Science Journal was used to measure the frequency of the sound produced from the bottle. It is a free iPhone app that is simple to use. It uses the microphone in the device to record the audio.

| Volume air/ml ± 0.5ml | Min frequency/Hz | Max frequency/Hz | Mean frequency/Hz | Uncertainty Δf/ ±Hz |
|---|---|---|---|---|
| 650 | 178,8 | 185,6 | 181,4 | 3,40 |
| 600 | 182,8 | 189,2 | 186,3 | 3,20 |
| 550 | 194,6 | 203,4 | 200,7 | 4,40 |
| 500 | 208,3 | 216,2 | 212,6 | 3,95 |
| 450 | 219,1 | 234,6 | 227,7 | 7,75 |
| 400 | 227,3 | 240,5 | 237,5 | 6,60 |
| 350 | 243,7 | 253,0 | 249,3 | 4,65 |
| 300 | 262,4 | 275,2 | 269,9 | 6,40 |
| 250 | 275,1 | 294,1 | 285,6 | 9,50 |
| 200 | 321,8 | 337,7 | 327,3 | 7,95 |
| 150 | 361,5 | 374,5 | 370,6 | 6,50 |

The app used to record the frequency, calculates the average frequency by adding several data points together, then dividing the sum on the amount of data points used. This gives the mean value. The app also picks out the highest and the lowest frequency recorded. These values were then plotted into Excel before being transferred to the table format.


This is an example of how the interface is and how the data could be collected. Marked with red circles are the datas for when there was 100ml of water in the bottle (550ml of air)

Since the min and max value of frequency is given for each water addition, the uncertainty can be calculated by using the formula below:

$$\Delta f = \frac{f_{max} - f_{min}}{2}$$

As stated in the theory, the frequency is inversely proportional to the square root of volume. Thus the volume also needed to be manipulated before plotting the graph.

| Volume air/ml ± 0.5ml | 1/sprt(volume air) |
|---|---|
| 650 | 0,0392 |
| 600 | 0,0408 |
| 550 | 0,0426 |
| 500 | 0,0447 |
| 450 | 0,0471 |
| 400 | 0,0500 |
| 350 | 0,0535 |
| 300 | 0,0577 |
| 250 | 0,0632 |
| 200 | 0,0707 |
| 150 | 0,0816 |

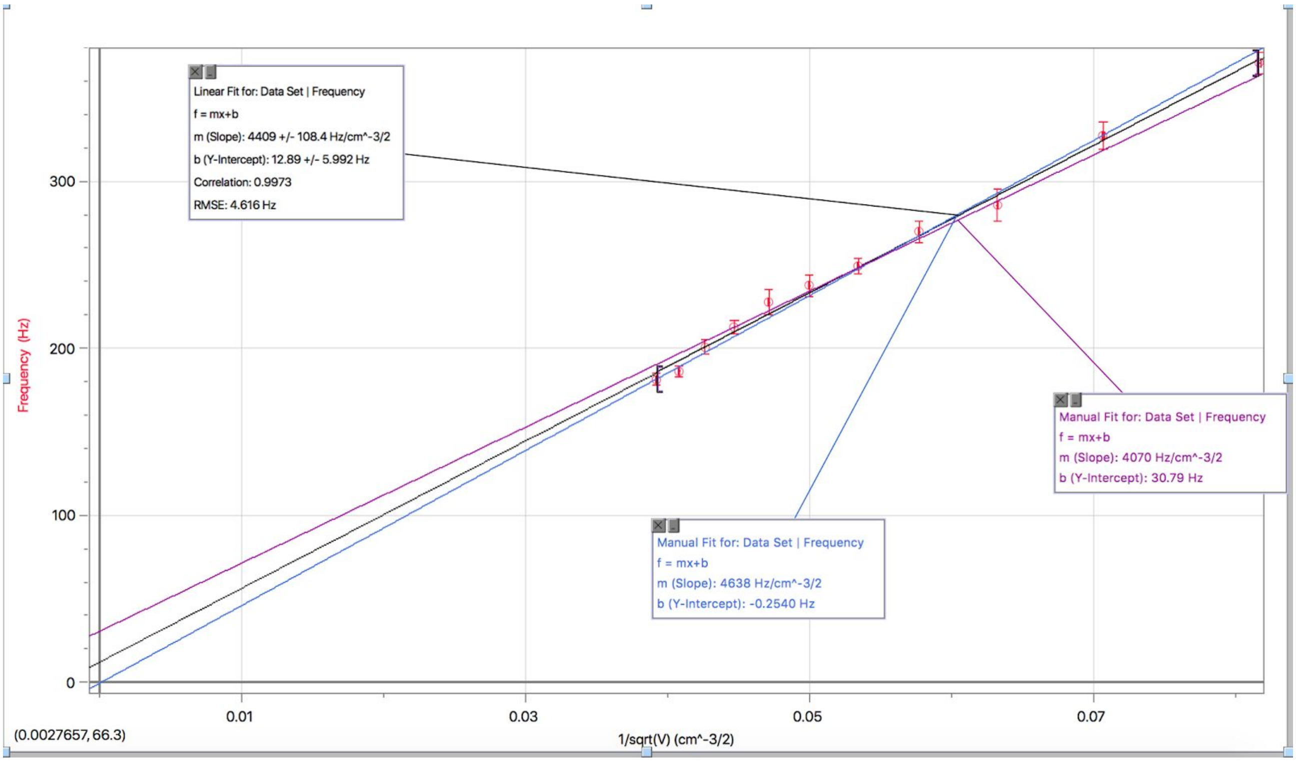
The uncertainty of 1/sqrt(volume) can also be calculated by using the same formula as for calculating the uncertainty of frequency

| Max volume air/ml | Min volume air/ml | 1/sqrt(max volume) | 1/sqrt(min volume) | Uncertainty 1/sqrt(volume)/± ml |
|---|---|---|---|---|
| 650,5 | 649,5 | 0,0392081 | 0,0392383 | -0,0000151 |
| 600,5 | 599,5 | 0,0408078 | 0,0408419 | -0,0000170 |
| 550,5 | 549,5 | 0,0426208 | 0,0426595 | -0,0000194 |
| 500,5 | 499,5 | 0,0446990 | 0,0447437 | -0,0000224 |
| 450,5 | 449,5 | 0,0471143 | 0,0471667 | -0,0000262 |
| 400,5 | 399,5 | 0,0499688 | 0,0500313 | -0,0000313 |
| 350,5 | 349,5 | 0,0534141 | 0,0534905 | -0,0000382 |
| 300,5 | 299,5 | 0,0576870 | 0,0577832 | -0,0000481 |
| 250,5 | 249,5 | 0,0631824 | 0,0633089 | -0,0000632 |

| | | | | |
|---|---|---|---|---|
| 200,5 | 199,5 | 0,0706225 | 0,0707992 | -0,0000884 |
| 150,5 | 149,5 | 0,0815139 | 0,0817861 | -0,0001361 |

As seen in the table above, the uncertainty for volume is very small. The minus in front of the numbers can be disregarded as the value is representing ±.

**<u>Graph</u>**




The gradient uncertainty (determined from the graph) is approximately:

$$\frac{4638-4070}{2} = \pm 284\ Hz\ cm^{\frac{-3}{2}}$$

The intercept uncertainty (determined from the graph) is approximately:

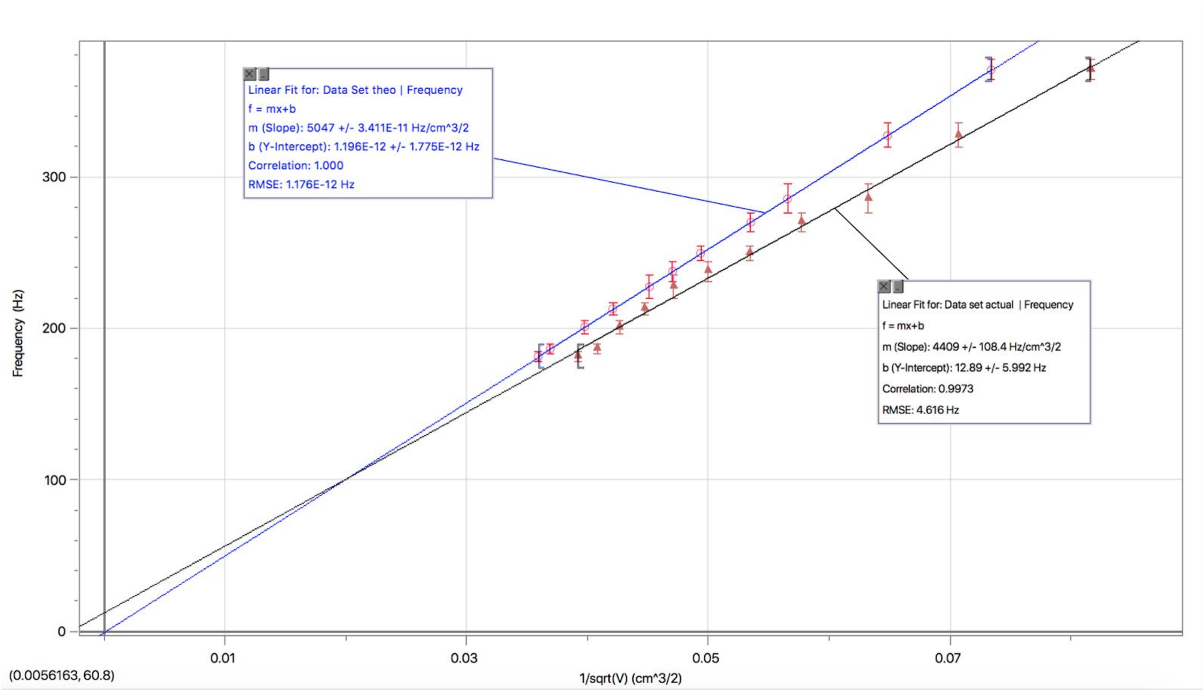
$$\frac{12.89-(-0.254)}{2} = \pm 6.572$$

**<u>Conclusion</u>**

From the graph we can conclude that the points form a linear graph, thus, the frequency is inversely proportional to the square root of the volume. Even though there is some uncertainty and errors involved, the points are relatively close to the linear best fit line. There is a clear relationship between the variables.

The y axis is intercepted by the best fit line at 12.89Hz which is relatively close to origo. Given the uncertainty of the intercept (±6.572) this shows that data is reliable and accurate.

The gradient of the actual measurements can be compared to the gradient of the theoretical Helmholtz resonance model. In the graph below, the same frequencies measured during the experiment are used in two different data sets; one with the corresponding theoretical value of volume, while the other corresponds to the actual volume. The theoretical data set goes through origo and the best fit line is colored blue. Experiment data set forms a black best fit line:



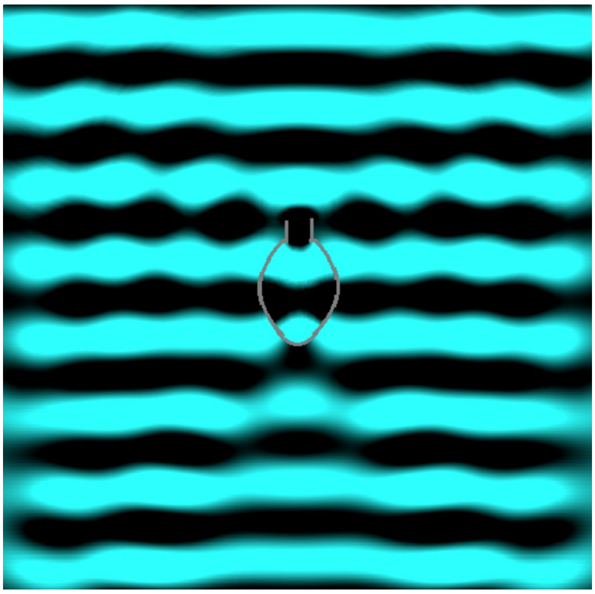
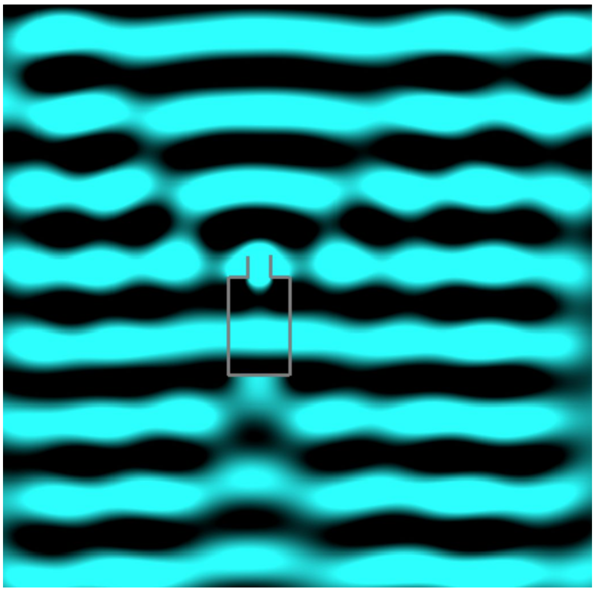

The deviation from theoretical line is relatively small. The gradient comparison verifies the the validity of Helmholtz equation.

## Evaluation and discussion

For this experiment, the results show good correlations with the hypothesis. The graph shows a linear correlation: my hypothesis expected this to happen. The spread of data from the best fit line is small and there is little random error. It is expected that the error bars touches the best fit line, but two of the error bars do not. Both error bars are at the lowest range for frequencies measured. These error bars are also smaller than the others. A possible explanation might be the microphone's ability to pick up lower frequencies, thus this can be classified as a systematic error. A way to get more accurate resonant frequencies would be to use a better microphone.

The best-fit line intercept with the y-axis is relatively close to the expected intercept of origo. It intercepts at 12.89Hz which, in a relative matter is very close to zero as the frequency increase is steep in the graph.

In my theory I stated that the geometric figure of the bottle might affect the frequency of sound. This is hard to measure in practice, but using Paul Falstad's online simulation I was able to do a qualitative test with simplified 2D bottles:

Due to limitations of the simulator, the volumetric size is out exactly the same, but still very close. In both pictures, the same source frequency is used. Both bottles resonate the same 3rd harmonic. The measured resonating frequency is the same (only negligible differences). However, the length of the neck and size of the opening still plays an important role in the emitted frequency. From this I can determine that the geometric figure (excluding port opening and neck length) of a bottle has insignificant or no correlation with the emitted frequency (the qualitative data is not precise enough for an exact conclusion).

The theoretical value of volume of air (using helmholtz resonance plus the correction value) can be compared to the actual value of volume of air:

| Theoretical value of air/ml | Actual volume of air/ml | Percentage error |
|---|---|---|
| 774,14 | 650 | 16,04 |
| 733,95 | 600 | 18,25 |
| 632,41 | 550 | 13,03 |
| 563,59 | 500 | 11,28 |
| 491,32 | 450 | 8,41 |
| 451,61 | 400 | 11,43 |
| 409,87 | 350 | 14,61 |
| 349,69 | 300 | 14,21 |
| 312,30 | 250 | 19,95 |
| 237,79 | 200 | 15,89 |
| 185,47 | 150 | 19,13 |

Even with the correction of Helmholtz resonance, there is a moderate percentage error. Another correction might have given more accurate data. Still, there is an clear correlation between the frequency and volume as shown in the graph. The original Helmholtz resonance is most likely limited ideal conditions, while for practical situations, the addition of correction will give a more precise data. I can conclude that the difference between theoretical and actual data is heavily reliant on the correction of l'. The borrowed correction greatly improved the theoretical data set, however, an investigation to deduce a better correction equation could close the percentage error even more.

The data collected was reasonable for this experiment, however, there are several ways I could have improved my method to get even better results. First of all, due to the lack of infinite resources, I could not get ahold of certain equipment that may have had a huge impact on the quality of data. Equipment such as acoustic foam and a better microphone could have reduced the impact of background noise and distortion.

Still, there are other areas of improvement that are within the reach of my capability. I could have collected more data to reduce the overall uncertainty, but the amount of data already used is quite good. There might also have been some uncertainty with the angle of the air stream. This could have affected the value of the correction model. It would be impossible for me to keep the consistency of air stream fixed. This might have affected the frequency recorded, however the total impact is unclear. A possible solution would be to not use a human variable, but rather a some type of fan with a concentrated direction (like a low powered jet stream).

## Works cited


helmholtz's volume resonator - Kamaljeeth Instrument (borrowed air pressure constant equation)
http://fisicaondemusica.unimore.it/Risuonatori_di_Helmholtz_en.html (image)
https://en.wikipedia.org/wiki/Helmholtz_resonance#/media/File:Helmholtz_resonator.jpg (image)
http://www.vibrationdata.com/Newsletters/January2004_NL.pdf (Correction suggestion)
http://falstad.com/ripple/ (2d ripple tank)
Physics HL - Chris Hamper - Second Edition - Pearson 2014 (quote/info)